

SCUOLA POLITECNICA E DELLE SCIENZE DI BASE

**FOUNDATIONS OF ROBOTICS**

**INGEGNERIA DELL’AUTOMAZIONE E ROBOTICA**

ANALYSIS AND CONTROL OF A SCARA ROBOT

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# Abstract

In the following chapters, the problem of controlling a 4-DOF SCARA manipulator will be addressed. The first and second chapters will deal with the derivation of the manipulator’s kinematic, as well as its velocity and force manipulability ellipsoids. Chapter 3 will then focus on the choice of a feasible operational space trajectory, which in Chapter 4 will be converted into the joint space via various CLIK algorithms. Finally, Chapter 5 will deal with the derivation of the manipulator’s dynamic model and the implementation of two joint space control strategies: robust and adaptive control.

# Kinematic analysis

## Denavit-Hartenberg

## The first step in building the robot’s model is always choosing the frames attached to the manipulator’s links. This can be done via the Denavit-Hartenberg convention (DH). Using DH, frames can be positioned as in figure, resulting in the set of parameters in the table.

Immagine che contiene diagramma, schizzo, Disegno tecnico, linea

Descrizione generata automaticamente

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Links | 𝑎𝑖 | 𝖯𝑖 | 𝑑𝑖 | 𝛼𝑖 |
| 0 | 0 | 0 | 1 | 0 |
| 1 | 0.5 | 𝖯1 | 0 | 0 |
| 2 | 0.5 | 𝖯2 | 0 | 𝜋 |
| 3 | 0 | 0 | 𝑑3 | 0 |
| 4 | 0 | 𝖯4 | 0 | 0 |

## Direct Kinematics and kinematic model

𝑥𝑒 = 𝑘(𝑞)

In this case we have:

𝑎1𝑐𝑜𝑠𝜃1 + 𝑎2cos (𝜃1 + 𝜃2)

𝑘(𝑞) = [ 𝑎1𝑠𝑖𝑛𝜃1 + 𝑎2sin (𝜃1 + 𝜃2) ]

𝑑0 − 𝑑3

𝜃1 + 𝜃2 − 𝜃4

where:

𝜃1

𝑞 = (𝜃2 )

𝑑3

𝜃3

is the joint coordinates array. Since an analytical expression for 𝑘(𝑞) is available, computing the analytical Jacobian is a fairly simple task. In this case, analytical Jacobian is represented by the following 4x4 matrix:

𝜕𝑥𝑒

𝖥𝑑𝑥1

I𝑑𝑞1

⋯ 𝑑𝑥11

𝑑𝑞4I

1 1 2 1 2

2 1 2

𝐽𝐴(𝑞) =

= I ⋮ ⋱ ⋮

I = ( )

𝜕𝑞

I𝑑𝑥4 [𝑑𝑞1

⋯ 𝑑𝑥4I

𝑑𝑞4]

Since our degrees of freedom let the system rotate just around z axis, geometric and analytical Jacobian are equivalent. So:

|  |  |  |  |
| --- | --- | --- | --- |
| −𝑎 𝑠𝑖𝑛𝜃 − 𝑎 sin (𝜃 + 𝜃 ) | −𝑎 sin (𝜃 + 𝜃 ) | 0 | 0 |
| 𝑎1𝑠𝑖𝑛𝜃1 + 𝑎2sin (𝜃1 + 𝜃2) 𝑎2cos (𝜃1 + 𝜃2) 0 0 | | | |
| 0  1 | 0  1 | −1  0 | 0  −1 |

𝐽(𝑞) = 𝐽𝐴(𝑞)

Jacobian represent the connection between velocities in joint space and operational space:

𝑣𝑒 = 𝐽(𝑞) 𝑞̇,

while Jacobian transpose provides the connection between end effector forces and joints torques:

𝑟 = 𝐽𝑇(𝑞) 𝛾𝑒

These equation enlightens kineto-statics duality.

# Manipulability analysis

Manipulabilty analysis allow to analyze velocities and forces in operational space that can be generate in a certain assigned configuration, through the use of manipulability ellipsoids in velocity and force.

Consider the set of joint velocities of constant unit norm::

𝑞̇𝑇𝑞̇ = 1

This equation describes the points on the surface of a sphere in the joint velocity space. Through differential kinematics, we obtain the quadratic form:

𝑣𝑇(𝐽(𝑞)𝐽𝑇(𝑞))−1𝑣𝑇 = 1

𝑒 𝑒

Ellipsoid’s form and orientation depend on (𝐽(𝑞)𝐽𝑇(𝑞))−1. Eigenvectors of this matrix define main axis directions, while eigenvalues define their dimensions.

In a dual way, in force’s scenario, one can consider the sphere in the space of joint torques:

𝑟𝑇𝑟 = 1

Which is mapped into the ellipsoid in the space of end effector forces:

𝛾𝑇(𝐽(𝑞)𝐽𝑇(𝑞))𝛾𝑇 = 1

𝑒 𝑒

As can be easily recognized, the core of the quadratic form is constituted by the inverse of the matrix core of the velocity ellipsoid.

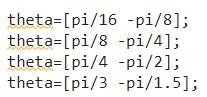
This feature leads to a notable result: principal axes of manipulability ellipsoid in force coincide with the principal axes of manipulability ellipsoid in velocity, while the dimensions of the respective axes are in inverse proportion. Therefore, according the concept of force-velocity duality, a direction along which good velocity manipulability is obtained, is a direction along which poor force manipulability is obtained, and vice versa.

The quantity:

𝜔(𝑞) = √𝑑𝑒𝑡(𝐽(𝑞) 𝐽𝑇(𝑞),

proportional to the volume of manipulability ellipsoid in force at a given posture, is a simple indicator of the distance between the chosen configuration and singular configurations. Notice how volume decreases as the manipulator assumes outstretched and retracted configurations.

Because of the SCARA robot’s simple structure, only the manipulator’s backbone (i.e. its first 2 links) is of interest. Therefore, a reduced Jacobian, composed of only few configurations of the Jacobian has to be considered. Indeed, for the purpose of manipulability analysis, the manipulator has been reduced to a 2- link planar arm. We now consider the following configurations:



The obtained results are:

For every configuration, manipulability measure has been computed:

### 𝑤(𝑞) = √det (𝐽(𝑞)𝐽(𝑞)𝑇)

Results are:

* Configuration 1: 𝜔 =
* Configuration 2: 𝜔 =
* Configuration 3: 𝜔 =
* Configuration 4: 𝜔 =0